Volatility Model Choice for Sub-Saharan Frontier Equity Markets - A Markov Regime Switching Bayesian Approach

Carl Hope Korkpoe  
University of Cape Coast, Ghana | ckorkpoe@ucc.edu.gh

Nathaniel Howard  
University of Cape Coast, Ghana | nhoward@ucc.edu.gh

Abstract

We adopt a granular approach to estimating the risk of equity returns in sub-Saharan African frontier equity markets under the assumption that, returns are influenced by developments in the underlying economy. Four countries were studied – Botswana, Ghana, Kenya and Nigeria. We found heterogeneity in the evolution of volatility across these markets and also that two-regime switching volatility models describe better the heteroscedastic returns generating processes in these markets using the deviance information criteria. We backtest the results to assess whether the models are a good fit for the data. We concluded that, the selected models are the most suitable for predicting the volatility of future returns in the markets studied.

Keywords: Regime-Switching, Bayesian Markov Chain Monte Carlo, Frontier Equity Markets, Business, Statistics

* This paper is the first extract of the ongoing PhD in Statistics thesis by the first author. The second author is the principal supervisor.
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1. Introduction

Investor interest in frontier equity markets of sub-Saharan Africa has been growing lately. This interest is topical in a growing finance and economics literature (Sukumaran, Gupta and Jithendranathan 2015; De Groot, Pang and Swinkels 2012; Quisenberry Jr and Griffith, 2010) and in International Finance Corporation (2013), the World Bank's private sector arm's newsletters for some time now. Bley and Saad (2012) suggest that, the principal motivation for this development is the low correlation of frontier market asset returns with their counterpart returns in the developed and emerging markets. However, perceived heightened risk is prevalent in the discussions related to investment and portfolio management activities in the frontier markets as a whole (Hassan, et al., 2003).

Much is known about volatility and the strategies for managing risks in investments in various assets in the developed and emerging markets. However, the same cannot be said for the smaller frontier markets which are ironically becoming important additions to portfolios of global investors seeking additional returns in the face of dwindling markets returns in the developed and emerging markets (Marshall, Nguyen and Visaltanachoti, 2013). The characteristics of these frontier markets are quite different from the emerging and developed market counterparts. For one thing, investors in frontier markets are starved of information about the markets they invest in (Sukumaran, Gupta and Jithendranathan, 2015). In equities markets, there are still problems of corporate governance of the listed firms and also insider market transactions, which are shrouded in secrecy (La Porta, et al. 2000, Klapper and Love, 2004). Besides, frontier economies are prone to bouts of uncertainty, foreign exchange crises, changing capital market regulations and at times political upheavals. One, therefore, will expect these to be transmitted to the broad asset markets as regime changes in market outcomes.

Externally, global developments influence the dynamics of economic activity in much of the developing world where some of these frontier markets are located. For long, a key factor that rolls markets in emerging and frontier markets is US monetary policy and the demand for commodities from the developed world and China. A surge in the value of the US dollar normally causes a fall in the equities of the smaller markets of the frontier economies as global investors rebalance their portfolio holdings away from such markets towards profitable opportunities in developed markets. In the process, there is a surge in volatility of frontier market equities. The aversion for frontier market equities is further strengthened if the developed country economies are growing robustly.

The environment, therefore, generating the market data is influenced by economic, social and political developments which are in a continuous flux across time. These should determine the magnitude and the associated fluctuations of the asset returns conditional on the regimes generating the data. Modeling the volatility of stock market returns using regime switching is therefore motivated by these developments in the underlying economy.

Volatility modeling incorporating regimes have been studied extensively across the developed and emerging markets in various asset classes starting with the pioneering efforts of Hamilton (1989). Subsequent to this, Hamilton and Susmel (1994) and Hamilton (1996) gave additional insights into the behavior of returns using extensive economic data. Other authors, for example, Hardy (2001) and Gray (1996) have utilized regime switching models in equities and interest rates respectively and they have concluded that such models provide a better fit to the data than single regime GARCH models. In the emerging markets, the first attempt at characterising the volatility of market returns with regime switching models was by Assoe (1998). He assigned changing government policies and reforms in capital markets as the main factors driving regime changes in emerging markets. Since then, a number of papers have appeared in the literature identifying regimes in the returns of various assets in emerging markets (Ozdemir and Akgul 2015; Van Gysen, Huang and Kruger 2013; Li, 2013).


There is plenty of evidence in the finance literature suggesting that multi-regime volatility models provide a better description for modelling asset returns. Lamoureux, and Lastrapes (1990) investigated the
performance of GARCH models in financial markets and concluded that they perform poorly in the face of changing regimes in the data. They attributed this to lack of fit of the models to the data. Single-regime volatility GARCH models are slow to react to changes in the returns structure that results from the sudden, albeit, inevitable changes that buffet the economy; hence the data generating process. Such changes in regimes are observed empirically. Regime switching models, thus, serve to capture the resulting rich heteroscedastic dynamics.

The clearest evidence of the presence of market regimes is the behaviour of the stock market during a recession vis-a-vis during an expansion in the economy. Standard GARCH models do not staircase risk across time, effectively discounting the empirical evidence of the behavior of equity prices in periods of recessions and expansions alternately. During recessions, nervous investors flee the equity markets to the safety of sovereign instruments. The reverse is seen in economic expansions. Thus, characterising the risks in the financial markets cannot be divorced from the underlying dynamics at play in the economy especially in measures of volatility, a pure financial construct for risk. Regime switching models are therefore seen as providing sound econometric explanation to the empirics of volatility in equity markets. Marcucci (2005) demonstrated the superiority of Markov switching GARCH models over all standard classes of GARCH models in use by examining the volatility of US stock markets returns.

Accurately estimating volatility in equity markets is important in more ways than one. Equities are now tied to many financial products in the market for assets. Thus, any jolt in equities roils all assets creating feedback loops which unsettle investors and spark more selling into falling markets, exacerbating volatility even further (Shleifer and Vishny, 2011). Models that provide the flexibility to adapt to changing market outcomes or regimes provide investors with tools to position trading strategies that avoid losses during market downturns. Single regime models lack this flexibility as a result of few parameters in their specification. This causes overfitting of the data (Cheng, Yu and Li, 2009) and predictably breakdowns in the face of regime changes in the underlying data generating process.

This study, therefore, seeks to identify appropriate regime switching models that correctly characterize the volatility dynamics of returns of selected sub-Saharan frontier market equities and also serve as the basis of forecasting risks in such markets. We used a sample of daily returns of the broad market indices of Ghana, Nigeria, Kenya and Botswana stock exchanges from January, 2011 to December, 2017 for our models. Using a number of popular regime switching models in the finance literature, we estimated the volatility of the returns of these indices as regime switching Markov volatility models. We used the Bayesian Monte Carlo simulation to get round the path dependency problem identified in Augustyniak (2014) and Hahn et al. (2010). We backtest the results for accuracy of the tail forecasts with value-at-risk methodologies. The following are our findings.

Sub-Saharan frontier equity markets are not homogeneous. Different regime-switching GARCH models describe the heteroscedastic behaviour of all the market returns and with varying tail behaviour for the respective markets. The data generation processes of the returns are characterized by fat-tails. This can be a source of profitable opportunities at the same time risk of extreme losses for the investing community.

We make two important contributions to the literature on volatility in sub-Saharan frontier equity markets. First, we fill the void in the literature and provide concrete evidence of volatility regimes of market outcomes in the equity markets of sub-Saharan Africa. Investors have a guiding model to use in quantifying the evolving heteroscedastic risk with regime changes in frontier equity markets. This is true particularly for active managers implementing volatility-sensitive trading strategies who might seek to profit from market gyrations. Secondly, the paper provides a volatility model for monitoring regime changes in the volatility of returns in the frontier markets studied. This is fundamental to investment and portfolio management in the information starved environment of sub-Saharan frontier equity markets.

The progression of the rest of the paper is as follows: Section two looks at the regime-switching GARCH models employed in the study. An explanation of the Bayesian methodology is provided in section three. We undertake the analysis of the data in section four and make a decision on the appropriate regime switching model that provides the best fit to our respective sample data. A summary of the findings goes into section five. We backtest our result to assess model fit and the ability to forecast accurately in section six. Section seven concludes the paper.

II. Model

We denote the market returns by \[ r_t \] where \[ r_t = \ln \left( \frac{p_t}{p_{t-1}} \right) \] is the log returns calculated from the closing prices \( p_t \) on day \( t \). Adapting the notation of Ardia et al. (2018), the evolution of the volatility of returns in regime \( k \) given the information set at a time \( t-1 \), \( F_{t-1} \), parameters \( \sigma^2_{k,t} \) and \( \varepsilon_{k,t} \), the Markov regime switching GARCH is generally stated as:

\[
\sigma^2_{k,t} = \omega_k + \alpha_k r^2_{t-1} + \beta_k \sigma^2_{k,t-1}. \tag{2}
\]
The parameters $\omega_k$, $\alpha_k$ and $\beta_k$ are constrained as $\omega_k > 0$, $\alpha_k > 0$ and $\beta_k \geq 0$ to ensure a positive variance. An additional constraint is $\alpha_k + \beta_k < 1$ to guarantee regime k piecewise covariance stationarity.

The EGARCH Model
The EGARCH model was proposed by Nelson (1991) to capture the leverage effect identified in Black (1976). The model is specified as:

\[
\ln(h_{t,t}) = \omega_k + \alpha_1 \ln(h_{t-1,t-1}) + \alpha_2 \ln(y_{t-1,t}) + \beta_1 \ln(h_{t-1,k-1}) + \beta_2 \ln(h_{t-1,k-1})
\]

where $E[\ln(h_{t-1,k-1})]$ is the expectation conditional on a given regime $k$. To ensure that a given regime $k$ is stationary, we impose the restriction $\beta_2 < 1$.

The GJR-GARCH Model
Glosten et al. (1993) proposed the GJR-GARCH model to capture the asymmetry stylized fact of financial time series. This model is stated as:

\[
\sigma^2_t = \omega + \alpha_1 I(y_{t-1,t} < 0)\sigma^2_{t-1} + \beta_1 \sigma^2_{t-1}
\]

where $I(.)$ an indicator function taking a value 1 if the condition is true and 0 otherwise.

The joint distribution of the regimes, $S_k$, is given by:

\[
p(S_1, S_2, \ldots, S_k) = p(S_1)p(S_2|S_1)\ldots p(S_k|S_1, S_2,\ldots, S_{k-1})
\]

which becomes

\[
p(S_1, S_2, \ldots, S_k) = p(S_1) \prod_{t=2}^{k} p(S_t|S_{t-1})
\]

as the first-order Markov model. For two-regime states, $S_1$ and $S_2$, we define the transition matrix $p_{ij}$ as:

\[
p(S_{t+1} = j|S_t = i) = p_{ij} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}
\]

III. Bayesian Parameter Estimation via MCMC
Let the parameter space be $\Phi = (\omega, \alpha, \beta, \sigma^2, \omega, \Pi_{11})$ and $S = (S_1, S_2, \ldots, S_k)$. We derive the posterior as joint distribution of the parameters conditional on the returns $r_t$. Invoking Bayes’ rule and the conditional probability property, we write the joint distribution for the posterior as follows:

\[
p(\Phi, S_1|r_2) \propto p(\Phi)p(S_1)p(r_1|\Phi, S_1) \propto p(\Phi)p(S_1)p(r_1|\Phi, S_1).
\]

The first term, $p(\Phi)$, on the right hand side is the prior specifying the hyperparameters of the distribution. This prior is expanded further as:

\[
p(\Phi) = (p(\omega)p(\alpha)p(\beta)p(\sigma^2)p(\omega, \alpha, \beta)p(\sigma^2)p(\omega, \alpha, \beta)p(\Pi_{11})).
\]

Using the assumption of independence, (9) becomes:

\[
p(\Phi) = p(\omega)p(\alpha)p(\beta)p(\sigma^2)p(\Pi_{11}).
\]

Das and Yoo (2004) and Kim and Nelson (1999) suggested normal priors for $\omega$, $\alpha$ and $\beta$. Ardia and Hoogerheide (2010) used truncated normal distributions to specify the priors for these GARCH parameters in order to maintain the constraints required for the variance to be positive. This approach is what we adopt in this paper. We specify an inverse gamma distribution for the variance, $\sigma^2$, as:

\[
\sigma^2 \sim IG(\alpha, \beta)
\]

where $\alpha$ and $\beta$ are the respective shape and scale parameters of the inverse gamma distribution.

The full conditional density of the posterior is complicated and unknown a priori, so we adopt the Metropolis-Hastings (Hastings 1970; Metropolis, et al., 1953) algorithm of the MCMC for our posterior simulation.

IV. Data and Analysis
Data for our analysis came from Botswana, Ghana, Kenya and Nigeria bourses. These countries were selected to reflect the regional groupings of west, east and southern African frontier equity markets based on the classification of MSCI Frontier Index (2019). Nigeria was added to the countries, because it has the most thriving equity market in sub-Saharan Africa classified as frontier. Our sample is made up of the respective broad market daily index from January 04, 2011 to December 29, 2017. Figure 1 plots the evolution of the levels of the indices.

![Figure 1. Evolution of Indices of the Stock Exchanges](image-url)
Figure 2. The Log-Returns of the Broad Market Indices

The returns of the four indices have been generally volatile during the sample period. Day-to-day returns of the BSI DCI, GSE and Nairobi have gyrated more wildly than the NSE. Volatility clusters characterize the returns of the GSE and Nairobi index returns compared to the NSE returns. On occasion, however, NSE has seen extreme outcomes. For example, mid-2016 saw the market return hitting close to -27%. The BSI DCI exhibits frequent turbulence during the period with negative returns mostly.

A summary of the statistics of the returns is provided in Table 1.

Table 1: Summary Statistics of the Returns of the Indices

<table>
<thead>
<tr>
<th>Index</th>
<th>Interquartile</th>
<th>Median</th>
<th>Lower Quartile</th>
<th>Upper Quartile</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSE</td>
<td>0.2010</td>
<td>1.3255</td>
<td>0.1831</td>
<td>3.1682</td>
<td>1.5460</td>
<td>5.5129</td>
</tr>
<tr>
<td>NSE</td>
<td>0.2022</td>
<td>1.0145</td>
<td>0.1817</td>
<td>3.2117</td>
<td>2.0034</td>
<td>8.1377</td>
</tr>
<tr>
<td>Nairobi</td>
<td>0.2850</td>
<td>1.3177</td>
<td>0.1849</td>
<td>3.3828</td>
<td>2.1569</td>
<td>4.5560</td>
</tr>
<tr>
<td>BSI DCI</td>
<td>0.3062</td>
<td>1.0127</td>
<td>0.1946</td>
<td>3.1659</td>
<td>0.0069</td>
<td>-0.1040</td>
</tr>
</tbody>
</table>

The results in Table 1 show varying degrees of deviation of the returns from normality. In particular, the tails of the distributions are fat with the severity exhibited by the returns from the NSE. Additionally, the returns from all the markets with the exception of the GSE are left skewed.

We plotted the returns to visually assess how far the returns deviate or otherwise conform to the Gaussian distribution. This is shown in Figure 3.

Figure 3. Histograms of Log-Returns with Kernels and Normal Density Curves Superimposed

In all cases, the returns seem to deviate from the normal density (in blue) superimposed on the distributions. Extreme outcomes are common. The returns for the NSE are within a narrow range, but with extreme outcomes. The tails exhibit heavier tails than normal in all cases with the severity of extremes much pronounced for the BSI DCI. This is confirmed by the summary statistics in Table 1.

A formal Shapiro-Wilk test for normality returned the W-statistics and the corresponding p-values shown in Table 2.

Table 2: Result of the Shapiro-Wilk Test

<table>
<thead>
<tr>
<th>Index</th>
<th>W-statistics</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSE</td>
<td>0.9057</td>
<td>&lt; 2.2e-16</td>
</tr>
<tr>
<td>NSE</td>
<td>0.76805</td>
<td>&lt; 2.2e-16</td>
</tr>
<tr>
<td>Nairobi</td>
<td>0.93099</td>
<td>&lt; 2.2e-16</td>
</tr>
<tr>
<td>BSI DCI</td>
<td>0.78162</td>
<td>&lt; 2.2e-16</td>
</tr>
</tbody>
</table>

Using a significant level at the conventional 5%, we reject the null hypothesis of each of the returns being normally distributed.

Markov switching models are built from GARCH primitives, so we tested for the presence of (G)ARCH effects using Engle’s LM test (Engle, 1982). Under this test, in finite samples, the null hypothesis states that the residuals of a return series exhibits no ARCH effects, against the alternative that ARCH effects are present in the residuals. The test results are summarized in Table 3.

Table 3: Engle’s LM Test for (G)ARCH Effects

<table>
<thead>
<tr>
<th>Index</th>
<th>χ²</th>
<th>Lags</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSE</td>
<td>170.78</td>
<td>12</td>
<td>&lt; 2.2e-16</td>
</tr>
<tr>
<td>NSE</td>
<td>458.94</td>
<td>12</td>
<td>&lt; 2.2e-16</td>
</tr>
<tr>
<td>Nairobi</td>
<td>221.95</td>
<td>12</td>
<td>&lt; 2.2e-16</td>
</tr>
<tr>
<td>BSE</td>
<td>20.64</td>
<td>12</td>
<td>0.056</td>
</tr>
</tbody>
</table>

From Table 3, there are clearly ARCH effects in the returns series of the GSE, NSE and Nairobi at the 5% significance level. The ARCH effects in BSI DCI returns are just borderline at 5% significant level.

We built the regime switching models for single- and two-state regime-switching GARCH models. The GARCH models considered are the normal GARCH, the EGARCH and GJR-GARCH for the skewed and student-t innovations. These are the GARCH models popular in the finance literature for volatility modeling. In all, we have twelve models to choose from. We employed the MSGARCH package developed by Ardia et al. (2016) using the R statistical language (R Core Team, 2018). The Deviance Information Criterion (DIC) served as a basis in selecting the best fitting GARCH model to the data. The results of the analyses are displayed in Table 4 and 5.

The respective selected models for the various stock exchanges with the minimum Deviance
Information Criteria (DIC) of Spiegelhalter et al. (2002) have been highlighted in Table 4 and 5.

### Table 4: DICs for Two-State Regime Switching GARCH Models

<table>
<thead>
<tr>
<th>Exchange</th>
<th>GARCH</th>
<th>GARCH</th>
<th>GARCH</th>
<th>GARCH</th>
<th>GARCH</th>
<th>GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ghana Stock Exchange</td>
<td>1841.726</td>
<td>1640.376</td>
<td>1826.856</td>
<td>1834.381</td>
<td>1854.245</td>
<td>1853.657</td>
</tr>
<tr>
<td>NSE</td>
<td>4424.137</td>
<td>4410.622</td>
<td>4465.106</td>
<td>4427.834</td>
<td>4443.174</td>
<td>4431.138</td>
</tr>
<tr>
<td>Nairobi</td>
<td>1008.030</td>
<td>1003.848</td>
<td>1000.940</td>
<td>1003.821</td>
<td>1006.159</td>
<td>1005.062</td>
</tr>
</tbody>
</table>

### Table 5: DICs for Single-State Regime Switching GARCH Models

<table>
<thead>
<tr>
<th>Exchange</th>
<th>GARCH</th>
<th>GARCH</th>
<th>GARCH</th>
<th>GARCH</th>
<th>GARCH</th>
<th>GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ghana Stock Exchange</td>
<td>1935.269</td>
<td>1914.725</td>
<td>1895.492</td>
<td>1907.981</td>
<td>1916.795</td>
<td>1913.018</td>
</tr>
<tr>
<td>NSE</td>
<td>4453.335</td>
<td>4461.851</td>
<td>4453.346</td>
<td>4423.084</td>
<td>4444.349</td>
<td>4451.398</td>
</tr>
<tr>
<td>Nairobi</td>
<td>2100.930</td>
<td>2092.215</td>
<td>2088.764</td>
<td>2088.777</td>
<td>2089.072</td>
<td>2087.688</td>
</tr>
<tr>
<td>Botswana</td>
<td>1933.082</td>
<td>1921.252</td>
<td>1917.387</td>
<td>1917.297</td>
<td>1916.170</td>
<td>1914.567</td>
</tr>
</tbody>
</table>

Overall, the 2-state regime-switching GARCH models fit the returns better than the single-regime models. For the aggregate market index of Ghana, the fitting model is the 2-state regime switching GJR\-GARCH(1,1) with skewed student-t innovations. This would suggest asymmetry in response to shocks. The NSE returns are best described by a 2-regime GJR\-GARCH(1,1) with student-t shocks. The Nairobi and Botswana bourses have returns during this sample period well described by 2-state regime switching EGARCH(1,1) with skewed student-t innovations. This shows the dominance of leverage effects in these markets.

### Model Diagnosis

Results of the posterior estimates are reported in Tables 6, 7, 8 and 9 for GSE, BSI, NSE and Nairobi respectively. We assess the efficiency of the sampling method using the relative numerical efficiency (RNE) (Geweke, 1989). There are no clear guidelines in literature on the cut-off for an efficient RNE. Values close to one are efficient while those close to zero are an indication of working with non-independent samples. From the Tables 6-9, the values of RNE are low but not zero and so some degree of efficiency is achieved in our sampling.
The GARCH error terms are $\alpha_{11} = 0.0519$ and $\beta_{12} = 0.2560$. The reaction of the conditional volatility to market shocks differs for the respective regimes. In the high volatility regime, investors react much more to market shocks than in the low volatility regime when market events are much more serene. This is expected naturally in the financial market as investors would react to any market news when they are jittery. The persistence of volatility as shown by the values of the GARCH lags $\beta_k$ also differs to both regimes. Volatility is much persistent in regime 1 ($\beta_1 = 0.9273$) than in regime 2 ($\beta_2 = 0.6762$). As pointed out by Alexander (2008), a $\beta$ above 0.9 is an indication that volatility takes a long time to die out following a market shock. Thus, the low regime volatility is persistent. This is in line with observations of thin and asynchronous trading on the GSE which effectively ensure that a given level of volatility is maintained for a long time. The unconditional volatility for regimes 1 and 2 are 3.01% and 13.06% respectively. The low volatility could be a reflection of lack of activity in most listed stocks.

The rate of convergence to the long term volatility for regime 1 ($\alpha_{11} + \frac{1}{2}\alpha_{21} + \beta_1 = 0.97925$) is slightly faster than that of regime 2 ($\alpha_{12} + \frac{1}{2}\alpha_{22} + \beta_2 = 0.93305$). Again, a low tail thickness value, $\xi_1 = 2.4927$, indicates that the distribution of the returns in regime 1 has thicker tails than regime 2 with $\xi_2 = 5.6411$. To summarize, both regimes are skewed ($\xi > 0$) with the tendency to remain in regime 1 rather dominant ($p_{11} \approx 49\%$). Regime 1 has a low reaction to past negative shocks with a high persistence in volatility. Regime 2 which is much more volatile with unconditional volatility of 18.5%, tends to be short lived ($p_{22} \approx 25\%$) with low volatility persistence.

**Nigeria Stock Exchange**

The volatility dynamics of the Nigeria Stock Exchange is characterized by two regimes, each with a different unconditional volatility of 12.92% and 52.43%. A dominant low regime has a probability of 87% and a high regime lasting approximately 10% of the time. The market shows a swift reaction to market shocks in the high volatility regime ($\alpha_{13} = 0.1427$) than in the low market regime ($\alpha_{11} = 0.0453$). Similar to the GSE, the persistence of the volatility indicated by the values of the GARCH lags $\beta_k$ also differs by regimes. Volatility is persistent in regime 1 ($\beta_1 = 0.6008$) compared to that in regime 2 ($\beta_2 = 0.2012$). Here, the values of this GARCH estimate is less than the threshold of 0.9 indicating that volatility takes relatively short time to abate following a market disturbance.

The rate of convergence to the long term volatility for regime 1 ($\alpha_{11} + \frac{1}{2}\alpha_{21} + \beta_1 = 0.73685$) is faster than that of regime 2 ($\alpha_{12} + \frac{1}{2}\alpha_{22} + \beta_2 = 0.44795$). The distribution of the returns in the high volatility regime exhibits heavier tails ($\nu_1 = 34.5793$) than the distribution in the low volatility regime ($\nu_1 = 56.2918$). We summarise the heteroscedastic characterization of the NSE as having a swift reaction to market shocks in regimes with a longer time for volatility to die out in regime 1. The low volatility regime is dominant with the high regime persisting only about 10% of the time.

**BSI DCI**

The volatility of the low regime of the BSI DCI index with an unconditional volatility of 10.49% is relatively longer (with a probability of $p_{11} = 0.2955$) than the high volatility regime (unconditional volatility of 23.74%) with a probability of $p_{13} = 0.1514$. The regimes alternate after short periods. This is confirmed in the graph in Figure 2. The GARCH error parameters for regime 1 and regime 2 are $\alpha_{13} = 0.0028$ and $\alpha_{23} = 0.0247$ respectively. This is below the threshold of 0.1 stated in Alexander (2008). Thus, the market has different reactions to negative past returns but, it is very insensitive to market events. This observation could be due to the asynchronous and thin trading effects in much of the frontier markets of sub-Saharan Africa with a small number of institutional investors and market analysts keeping an eye on market developments. The rate of convergence of the volatility to its long term level is $\alpha_{13} + \frac{1}{2}\alpha_{23} + \beta_1 = 0.8858$ and $\alpha_{12} + \frac{1}{2}\alpha_{22} + \beta_2 = 0.92245$, respectively, for the low and high regimes. The term structure of volatility for regime 2 is therefore relatively flat compared to regime 1.

The tails of both regimes ($\nu_1 = 2.1001$ and $\nu_2 = 2.1$) are fat. This is supported by the histogram in Figure 3 of the returns of the BSI DCI index. Investors can therefore make rich pickings in the market with careful selection of equities. The shape of the distribution in regime 1 is relatively skewed ($\xi_1 = 1.6521$ in Table 9: Estimates of 2-State Regime E-GARCH with Skewed Student-t Innovations – Nairobi

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Mean</th>
<th>SD</th>
<th>SE</th>
<th>TSSK</th>
<th>RNE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{01}$</td>
<td>0.3780</td>
<td>0.0826</td>
<td>0.0018</td>
<td>0.004</td>
<td>0.2131</td>
</tr>
<tr>
<td>$\alpha_{11}$</td>
<td>0.5835</td>
<td>0.0975</td>
<td>0.0017</td>
<td>0.0031</td>
<td>0.2223</td>
</tr>
<tr>
<td>$\alpha_{24}$</td>
<td>0.1666</td>
<td>0.0533</td>
<td>0.0012</td>
<td>0.0026</td>
<td>0.2164</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.3108</td>
<td>0.0738</td>
<td>0.0017</td>
<td>0.004</td>
<td>0.1723</td>
</tr>
<tr>
<td>$\nu_1$</td>
<td>4.5252</td>
<td>0.7915</td>
<td>0.0164</td>
<td>0.047</td>
<td>0.1212</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>0.8803</td>
<td>0.0849</td>
<td>0.0008</td>
<td>0.0018</td>
<td>0.1902</td>
</tr>
<tr>
<td>$\alpha_{02}$</td>
<td>0.5342</td>
<td>0.2600</td>
<td>0.0058</td>
<td>0.6174</td>
<td>0.1128</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>0.4854</td>
<td>0.4253</td>
<td>0.0085</td>
<td>0.6443</td>
<td>0.0401</td>
</tr>
<tr>
<td>$\nu_2$</td>
<td>2.1001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.6401</td>
<td>0.1354</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>1.6521</td>
<td>0.1825</td>
<td>0.0041</td>
<td>0.6171</td>
<td>0.0569</td>
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<tr>
<td>$\mu_{11}$</td>
<td>0.981</td>
<td>0.0063</td>
<td>0.0001</td>
<td>0.6003</td>
<td>0.1634</td>
</tr>
<tr>
<td>$\mu_{21}$</td>
<td>0.996</td>
<td>0.0027</td>
<td>0.0006</td>
<td>0.6003</td>
<td>0.1114</td>
</tr>
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</table>
12.6066) compared with the almost symmetrical shape of regime 2 ($\xi_2 = 1.0532$).

**Nairobi Stock Exchange**

The volatility of the market index in Nairobi in both regimes is very sensitive to past market events ($\alpha_{12} = 0.5835, \alpha_{22} = 0.4854$). This might suggest an active market with lots of market analysts and seasoned or institutional investors. Unconditional volatility of regimes 1 and 2 respectively is 11.23% and 29.52%. The distributions of returns are heavy-tailed in both regimes, suggesting the possibility of extreme returns for either regime. The volatility persistence across both regimes are similar, $\alpha_{11} + \frac{1}{2} \alpha_{21} + \beta_1 = 0.9776$ and $\alpha_{12} + \frac{1}{2} \alpha_{22} + \beta_2 = 0.94175$. These values suggest that, volatility takes a long time to die out following a market jolt. The shape of the distribution parameters, $\xi_1 = 0.8803$ and $\xi_2 = 1.6521$ respectively for regimes 1 and 2, suggest slight asymmetries in the distributions. In their work, Fernandez and Steel (1998) suggest $\xi = 0$ for a symmetrical distribution.

The evolution of the conditional volatility for the four exchanges during the sample period is shown in Figure 4.

![Figure 4. Conditional Volatility of the Index Returns of the Four Sub-Saharan Equities](image)

For each of these countries, volatility surges are sharp, but revert quickly to moderate levels. The volatility uptick in most cases is clearly due to investor nervousness about developments in the underlying economy. For the most part, the effects of thin and asynchronous trading observed by Mlambo and Biekpe (2005) in most African equities is prevalent as seen in the dominance of low volatility regimes across the countries studied.

**VI. Backtesting the Results**

We checked our results out-of-sample as a precaution against overfitting of the models to the data. Complex statistical models are notoriously prone to overfitting and perform poorly out-of-sample, where they are most relevant. Casley and Talbot (2007) discuss extensively the problems particularly with Bayesian models in machine learning. We aim to assess the robustness of our models in predicting correctly the value-at-risk at the standard 5% quantile. We utilized the conditional coverage test (CC) of Christoffersen (1998) and the dynamic quantile (DQ) test of Engle and Manganelli (2004). The CC test jointly tests for the independence (Kupiec 1995) and unconditional coverage (Christoffersen 1998) provided that, the VaR exceedances are independent and non-autocorrelated. Thus, the null hypothesis is that, exceedances are independent and are also derived from the conditional and unconditional coverage test statistics. The joint statistic has a $\chi^2$ distribution with one degree of freedom. The dynamic quantile (DQ) test of Engle and Manganelli for backtesting VaR models is a regression of the hit sequence, $l_1$, on a set of explanatory variables including the VaR quantiles and a finite number of lagged hits. The dynamic quantile test provides an overall goodness-of-fit test of the models under consideration.

We generated 700 samples ex-post for the 0.05 percentile of the returns from the model. The test covered the chosen regime switching model and its corresponding non-switching counterpart for each of the stock markets studied. The results are shown in Table 10.

<table>
<thead>
<tr>
<th>Country</th>
<th>CC p-value</th>
<th>DQ p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>OSE</td>
<td>0.385839</td>
<td>0.7187228</td>
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<tr>
<td>DQ</td>
<td>0.0918936</td>
<td>0.1127054</td>
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<td>NSE</td>
<td>1.42382E-05</td>
<td>0.0005318</td>
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<tr>
<td>DQ</td>
<td>0.00096772</td>
<td>0.0029586</td>
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<td>Nairobi</td>
<td>0.524988</td>
<td>0.3982259</td>
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<tr>
<td>DQ</td>
<td>0.3181468</td>
<td>0.525648</td>
</tr>
<tr>
<td>BSI DQ</td>
<td>0.002733075</td>
<td>0.105842566</td>
</tr>
<tr>
<td>DQ</td>
<td>2.23E-07</td>
<td>4.94E-04</td>
</tr>
</tbody>
</table>

Table 10: Results of Conditional Coverage and Dynamic Quantile Tests

Results show a better model fit for the two-regime switching models for all exchanges, except the conditional coverage of Nairobi. The p-values of the DQ for the two-regime switching models are consistently higher than the single regime models. The conflicting finding of Nairobi has been discussed in the finance literature. Komunjer (2013) discussed at length the performance of these VaR tests and concludes that the dynamic quantile test by relying on out-of-sample simulations instead of the in-sample tests on which conditional coverage is based is a far more powerful test. We therefore conclude that, the two-regime Markov switching EGARCH for Nairobi is a better fit to the returns.

**VII. Conclusion**

Financial markets are subject to the developments in the underlying economy, which ultimately transmits to regime changes in returns. As it is, single regime models do not track closely the changing variability between bull and bear markets.
observed in equity returns. Risk dynamics do change in response to the economies in which markets operate. For the sub-Saharan African equity markets with a dearth of market information to aid decision-making (Enisan and Olufisayo 2009), keeping an eye on the developments in the broader economy together with how this affects market returns should provide competitive advantage to investors. Estimating volatility using regime switching informs investing by using risk estimates, which fairly reflect existing market conditions through the responsiveness of the estimating models to risk.

As our study shows, sub-Saharan Africa frontier equity markets are far from homogenous. Different dynamics, be it social, political or economic, are the driving forces behind the financial markets in these countries. Single regime models are likely to under- or over-estimate risk during periods of high or low volatility respectively. Therefore, a granular approach to estimating volatility that we have done for these sub-Saharan frontier equity markets seem the most rational way market actors can fine-tune their investment strategies to avoid losses during high volatility periods. All the markets studied show clear regimes in market volatility with different statistical properties.

Again, it is clear that though these markets are classified in the same bracket of frontier markets, different dynamics are driving their heteroscedastic evolution. Different heteroskedastic functions with varied tail behavior described the return data of Ghana, Nigeria, Nairobi and Botswana stock indices with observed asymmetric and leverage effects in the markets. Investors therefore have to be alert to generalizations that most likely will not fit all markets. Optimal risk-based allocation of investing funds based on volatility regimes across these markets adds to the benefits of diversification in equity trading in sub-Saharan African equity markets. In addition, any future risk-based capital allocation such as for Value-at-risk or expected tail loss that is regime aware based on the findings in this study should lead to prudent risk management and control. In all the markets under study, the low volatility regime dominates, leaving brief spells of market turbulence. This will provide investors with the information to lean against the winds during brief market gyrations.

References


